# Adiabatic potential energy curves of  $Li<sub>2</sub>$

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### ABSTRACT

Adiabatic potential energy curves for twenty six low-lying electronic states of  $Li<sub>2</sub>$  dimer have been computed in the large range of internuclear distances  $(3.2 a_0 \le R \le 80 a_0)$ . Four singlet states  $(4^1\Sigma_u^+, 2^1\Pi_u, 2^1\Pi_g$  and  $1^1\Delta_g$ ) and four triplet states  $(5^3\Sigma_u^+, 2^3\Pi_u, 2^3\Pi_g$  and  $1^3\Delta_u$ ) are presented for the first time. In the calculations, the complete-active-space multi-configuration self-consistent-field (CASSCF) method has been used. The corepolarization potential (CPP) has been also added to the hamiltonian. A good agreement with previous theoretical and experimental results has been obtained.

**Keywords:** Adiabatic potential energy curves,  $Li_2$  dimer, cold molecules

## 1. INTRODUCTION

The  $Li<sub>2</sub>$  molecule, for many years, has attracted attention of experimentalists and theoreticians. It is the second, just after  $H_2$ , smallest stable homonuclear molecule.  $Li_2$  has been studied by means of spectroscopical methods and a large number of electronic states has been tabulated. As an example, it can be given the recent experiment performed in Orsay<sup>1</sup> providing a better description of the  $1<sup>1</sup>\Pi_u$  state which correlates to  $2s + 2p$  asymptote. Here, the lithium molecules were produced by molecular beam and next they were excited by a tunable dye laser. Two kinds of measurements were performed: first, several hundred transitions to the ground  $1^1\Sigma_g^+$  state were accurately measured and second, the dissociation rates in quasibond levels were determined. In turn Pashov et al.<sup>2</sup> and Jastrzębski *et al.*<sup>3</sup> reported an accurate potential energy curves of  $3^{1}\Sigma_{g}^{+}$  and  $4\Sigma_{g}^{+}$  states. Their method was based on the inverted perturbation approach which reproduced positions of the experimental energy levels for all three isotopomers of  $Li<sub>2</sub>$ . We want to underline the recent experiments on photoassociative spectroscopy of long-range states of ultracold lithium dimers (Abraham et al.)<sup>4</sup>

There have also been several theoretical studies concerning the ground and excited states. We note the first accurate ab initio calculations of the ground state reported by Das and Wahl.<sup>5</sup> Next, we have to mention results of a few excited states obtained by Kutzelnigg et  $al$ .<sup>6</sup> Two very extensive calculations using the multi-configuration self-consistent-field (MC SCF) were performed by Olson and Konowalow<sup>7</sup> and next by Schmidt-Mink et al.<sup>8</sup> We compare our theoretical results with the both last approaches.

Very recently, there were also intensive experimental studies on the Bose-Einstein condensation of the  $Li<sub>2</sub>$ molecules in an optical trap (for example: Jochim et al.<sup>9</sup> and Bartenstein et al.<sup>10</sup>) and on fermionic pairing in an ultracold two-component gas of the Li atoms (for example: Chin et al.<sup>11</sup> and Jochim et al.<sup>12</sup>). Both experiments require high lying adiabatic energy curves, which nowadays are only partly available.

The aim of our theoretical approach is to proof validity of other methods based on the MOLPRO<sup>13</sup> program package in order to obtain high lying potential curves of the  $Li<sub>2</sub>$  dimer. In our calculations we use the complete-active-space multi-configuration self-consistent-field (CASSCF) method. Core electrons are treated by the effective-core potential (ECP) with the additional basis for the d electrons. Correlation is taken into account by employing the core-polarization potential (CPP). After CASSCF the configuration interaction (CI) calculations are performed. The spin-orbit coupling is not taken into account.

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## 2. THEORETICAL METHOD

We solve the Schrödinger equation, which in the Born-Oppenheimer approximation, can be written in the following form

$$
(H_1 + H_2 + V_{12})\Psi_{\kappa}(\mathbf{r}, \mathbf{R}) = E_{\kappa}(R)\Psi_{\kappa}(\mathbf{r}, \mathbf{R}),
$$

when  $H_1$  and  $H_2$  are the atomic hamiltonians of separated lithium atoms,  $V_{12}$  is the interaction between these two atoms,  $E_{\kappa}(R)$  is the adiabatic energy in the  $\kappa^{th}$  state which is described by the wavefunction  $\Psi_{\kappa}(\mathbf{r}, \mathbf{R})$ . Here, the  $\bf{r}$  are coordinates of all electrons of considered system and  $\bf{R}$  is the position vector giving the position of the first atom in relation to the second atom. In our calculations only the valence electrons of the lithium atoms are treated explicitly, what means that we consider effectively the two-electron system. The core electrons of Li atoms are represented by l-depended pseudopotential  $ECP2SDF<sup>14</sup>$ . The basis for the s and p orbitals which comes with this potential is enlarged by functions for d orbitals given by P. Feller<sup>15</sup> and assigned by CC-PV5Z. Additionally, our basis set was augmented by nine short range correlation functions and five diffuse functions. For details see Table 1. To account for the valence-core correlation the core-polarization potential is used. Detailed formulas can be found for instance in Czuchaj  $\it{et~al.}$  papers.  $^{16-18}$ 



Table 1. The basis set for s, p and d orbitals with augmented functions (bold numbers).

We check the quality of our basis set performing the CI calculation for the ground and excited states of isolated lithium atom. The results are given in Table 2. The agreement with experimental values is quite reasonable.The potential energy curves are calculated with multi-reference CASSCF calculation followed by CI calculation.

Table 2. Comparison asymptote energy with other theoretical and experimental results. Energies are shown in  $cm^{-1}$ units.

	Present work	Schmidt-Mink <sup>8</sup>	Bashkin <sup>19</sup>
$2s + 2p$	14916	14938	14904
$2s + 3s$	27201	27209	27206
$2p + 2p$	29822	29876	29808
$2s + 3p$	30931	31078	30925
$2s + 3d$	32113		31283

#### 3. RESULTS AND DISCUSSION

Calculation of the adiabatic energy curves are performed for the internuclear separation R in the range from  $3.2a_0$  to  $80a_0$  with the various step, according to the internuclear distance. Results for all singlet and triplet states are shown respectively in Fig. 1 and Fig. 2. The states  $5^3\Sigma_u^+$ ,  $4^1\Sigma_u^+$ ,  $2^1\Pi_u$ ,  $2^1\Pi_g$ ,  $2^3\Pi_u$ ,  $2^3\Pi_g$ ,  $1^1\Delta_g$ and  $1^{3}\Delta_{u}$  are calculated for the first time and were not known previously. In Fig. 3, we compare our ground and the first excited molecular state correlating to ground state of lithium atoms with the theoretical results of Olson and Konowalow<sup>20, 21</sup> and Schmidt-Mink et  $al$ <sup>8</sup> As one might see, the agreement is excellent. We make comparisons with potential curves derived from experiments. In Fig. 4, we show comparison with results derived from experiment by Bouloufa et  $al$ <sup>1</sup> and theoretical results of Schmidt-Mink et  $al$ <sup>8</sup> Once again we note the almost excellent agreement. In turn in Fig. 5, we display the comparison with two states of symmetry  $\Sigma_g^+$ derived from experiment by Pashov *et al.*<sup>2</sup> and Jastrzębski *et al.*<sup>3</sup> Here, in the case of  $4^{1}\Sigma_{g}^{+}$  state once again the agreement is excellent, while in the case of the lower  $3^{1}\Sigma_{g}^{+}$  state, correlating to  $2s + 3s$  atomic energy, we find the disagreement for small internuclear distance.

## 4. CONCLUSION

We provide previously unknown adiabatic potential curves. These are  $5^3\Sigma_u^+$ ,  $4^1\Sigma_u^+$ ,  $2^1\Pi_u$ ,  $2^1\Pi_g$ ,  $2^3\Pi_u$ ,  $2^3\Pi_g$ ,  $1<sup>1</sup>\Delta_g$  and  $1<sup>3</sup>\Delta_u$ . We believe in quality of our results, since comparison with chosen theoretical curves given by other authors is excellent. Also the agreement with the curves derived from spectroscopical measurements is excellent except the  $3^{1}\Sigma_{g}^{+}$  state, where in the repulsive part of the potential curve some disagreement is noted.

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## REFERENCES

- 1. N. Bouloufa, P. Cacciani, R. Vetter, A. Yiannopoulou, F. Martin and A. J. Ross, "A full description of the potential curve of the  $B^1\Pi_u$  state of <sup>7</sup>Li<sub>2</sub>," J. Chem. Phys. **114**, pp. 8445–8458, 2001.
- 2. A. Pashov, W. Jastrzębski and P. Kowalczyk, "The  $Li_2 F^{1} \Sigma_g^+$  "shelf" state: Accurate potential energy curve based on the inverted perturbation approach," J. Chem. Phys. 113, pp. 6624–6627, 2000.
- 3. W. Jastrzębski, A. Pashov and P. Kowalczyk, "The  $E^1\Sigma^+_g$  state of lithium dimer revised," J. Chem. Phys. 114, pp. 10725–10727, 2001.
- 4. E. R. I. Abraham, N. W. M. Ritchie, W. I. McAlexander and R. G. Hulet, "Photoassociative spectroscopy of long-range states of ultracold  ${}^6Li_2$  and  ${}^7Li_2$ ," J. Chem. Phys. 103, pp. 7773–7778, 1995.
- 5. G. Das and A. C. Wahl, "Extended Hartree-Fock wavefunctions: optimized valence configurations for  $H_2$ and  $Li_2$ , optimized double configurations for  $F_2$ ," J. Chem. Phys. 44, pp. 87–96, 1966.
- 6. W. Kutzelnigg, V. Staemmler and M. Gélus, "Potential curve of the lowest triplet state of  $Li_2$ ," Chem. Phys. Letters 13, pp.  $496-500$ , 1972.
- 7. M. L. Olson and D. D. Konowalow, "The potential energy curve for the  $B^1\Pi_u$  stete of  $Li_2$ ," Chem. Phys. Letters 39, pp. 281–284, 1976.
- 8. I. Schmidt-Mink, W. Müller and W. Meyer, "Ground- and excited-state properties of  $Li_2$  and  $Li_2^+$  from ab initio calculations with effective core polariztion potentials," Chem. Phys. 92, pp. 263–285, 1985.
- 9. S. Jochim, M. Bartenstein, A. Altmeyer, G. Hendl, S. Riedl, C. Chin, J. Hecker Denschlag and R. Grimm, "Bose-Einstein condensation of molecules," Science 302, pp. 2101–2103, 2003.
- 10. M. Bartenstein, A. Altmeyer, S. Riedl, S. Jochim, C. Chin, J. Hecker Denschlag, and R. Grimm, "Crossover from a molecular Bose-Einstein condensate to a degenerate Fermi gas," Phys. Rev. Lett. 92, pp. 120401– 120405, 2004.
- 11. C. Chin, M. Bartenstein, A. Altmeyer, S. Riedl, S. Jochim, J. Hecker Denschlag and R. Grimm, "Observation of the pairing gap in a strongly interacting Fermi gas," Science 305, pp. 1128–1130, 2004.
- 12. S. Jochim, M. Bartenstein, A. Altmeyer, G. Hendl, C. Chin, J. Hecker Denschlag and R. Grimm, "Pure gas of optically trapped molecules created from fermionic atoms," Phys. Rev. Lett. 91, pp. 240402–240405, 2003.

13. MOLPRO is a package of ab initio programs written by H. J. Werner and P. J. Knowles with contributions from R. D. Amos, A. Berning, D. L. Cooper, M. J. O. Deegan, A. J. Dobbyn, F. Eckert, C. Hampel, G. Hetzer, T. Leininger, R. Lindh, A. W. Lloyd, W. Meyer, M. E. Mura, A. Nicklass, P. Palmieri, K. Peterson, R. Pitzer, P. Pulay, G. Rauhut, M. Schütz, H. Stoll, A. J. Stone and T. Thorsteinsson.

H. J. Werner and P. J. Knowles, "A second order multiconfiguration SCF procedure with optimum convergence," J. Chem. Phys. 82, pp. 5053–5063, 1985.

P. J. Knowles and H. J. Werner, "An efficient second-order MC SCF method for long configuration expansions," Chem. Phys. Letters 115, pp. 259–267, 1985.

A. Berning, M. Schweizer, H. J. Werner, P. J. Knowles and P. Palmieri, "Spin-orbit matrix elements for internally contracted multireference configuration interaction wavefunctions," Mol.Phys. 98, pp. 1823–1833, 2000.

- 14. P. Fuentealba, H. Preuss, H. Stoll and L. Von Szentply, "A proper account of core-polarization with pseudopotentials: single valence-electron alkali compounds," Chem. Phys. Letters 89, pp. 418–422, 1982.
- 15. private communications, as cited in MOLPRO manual.<sup>13</sup>
- 16. E. Czuchaj, M. Krosnicki and H. Stoll, "Ab initio calculations for the potential curves and spin-orbit coupling of  $Mg_2$ ," Theor. Chem. Acc. 107, pp. 27–32, 2001.
- 17. E. Czuchaj, M. Krosnicki and H. Stoll, "Valence ab initio calculation of the potential energy curves for the Sr<sup>2</sup> dimer," Chem. Phys. Letters 371, pp. 401–409, 2003.
- 18. E. Czuchaj, M. Krosnicki and H. Stoll, "Valence ab initio calculation of the potential-energy curves for the  $Ca_2$  dimer," *Theor. Chem. Acc.* **110**, pp. 28–33, 2003.
- 19. S. Bashkin and J. O. Stoner Jr., "Atomic energy levels and grotian diagrams," Vol. 1, North-Holland, Amsterdam, 1975.
- 20. M. L. Olson and D. D. Konowalow, "Accurate potential energy curves for the  ${}^{3}\Sigma_{u}^{+}$  and  $b{}^{3}\Sigma_{g}^{+}$  states of  $Li_{2}$ ," Chem. Phys. 21, pp. 393–399, 1977.
- 21. D. D. Konowalow and M. L. Olson, "The electronic sructure and spectra of the  $X^1\Sigma_g^+$  and  $A^1\Sigma_u^+$  states of  $Li_2$ ," J. Chem. Phys. 71, pp. 450–457, 1979.



Figure 1. Adiabatic potential energy curves for ground and excited singlet states of the  $Li<sub>2</sub>$  molecule correlating to the  $2s + 2s$ ,  $2s + 2p$ ,  $2s + 3s$ ,  $2p + 2p$ ,  $2s + 3p$  and  $2s + 3d$  asymptotes.



Figure 2. Adiabatic potential energy curves for excited triplet states of the  $Li_2$  molecule correlating to the  $2s + 2s$ ,  $2s + 2p$ ,  $2s + 3s$ ,  $2p + 2p$  and  $2s + 3p$  asymptotes.



Figure 3. Comparison the ground and the first excited molecular state correlating to  $2s + 2s$  asymptote with the theoretical results of Olson and Konowalow<sup>20, 21</sup> and Schmidt-Mink *et al.*<sup>8</sup>



Figure 4. Comparison  $B(1)^1\Pi_u$  state correlating to  $2s+2p$  asymptote with results derived from experiment by Bouloufa et  $al$ <sup>1</sup> and theoretical results of Schmidt-Mink et  $al$ <sup>8</sup>



**Figure 5.** Comparison  $E(3)^{1}\Sigma_{g}^{+}$  and  $F(4)^{1}\Sigma_{g}^{+}$  states correlating respectively to  $2s + 3s$  and  $2p + 2p$  asymptotes with potential curves derived from experiment by Jastrzebski *et al.*<sup>3</sup> and Pashov *et al.*<sup>2</sup>

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